Identifying Significant Design Factors in Robost Product Designs

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WP # 3723 September 1994

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September 1, 1994

Abstract

In the screening phase of robust product design, investigators often need to identify most significant design factors among a large number of candidates. The significant design factors considered to be further investigated include those which either have significant main effects or(and) are important to eliminate variation due to uncontrollable environmental conditions. The significance of main effects of design factors demonstrate the contributions of the factor effects to the system response. The significance of design×environment interactions provide a measure of the robustness of design factors to environmental conditions.

In this paper we focus on estimating main effects of design factors and design×environment interactions. We assume that there may be some significant design×design and environment×environment interactions. However, we assume, as is often realistic, that all three or higher order interactions are negligible. We develop block cross-array strategy to construct a series of desired designs. Although the table given in our paper provides experiment designs of up to fifteen design factors and fifteen environment factors, it is easy to generalize the result for any size experiment system. The proposed design estimates main effects of design factors free from interactions and is superior to the cross-array design recommended by Dr. Taguchi.

* The research is partially sponsored by LFM Program, MIT Sloan School.

1 Introduction

1.1 Screening Analysis in Robust Design

In the traditional design of experiment analysis, screening designs are usually high fractional factorial designs. They are used for studying a large number of design factors[1] at the initial stage of experiment investigation. Plackett and Burman (1946) (P-B designs) and 2^{n-p}_{III} designs are typical low resolution designs for the screening purpose (Box and Meyer (1993); Hamada and Wu (1992); Dey (1985); Box and Meyer (1985); Box and Hunter (1961)). These resolution III designs provide unbiased estimation of main effects only when interactions are negligible. When some two-factor interactions are present, resolution IV designs are usually used to estimate main effects. The design factors with relatively significant main effects are chosen to be further investigated.

In the context of robust design, there is one more objective than merely looking for design factors with significant main effects. The general idea of robust design is to find the design factors which bring the response to the desired target and the design factors which influence the performance measure and can be used to minimize the variation caused by environmental sources. Therefore, the main objective is to select design factors either with relatively significant main effects and/or sensitivity to the uncontrollable environment conditions.

Signal-to-noise ratio (SN ratio) proposed by Taguchi (1986) is one of the measures used to study the robustness of design factors to the environment conditions. The significance of design$\times$environment interactions is another appropriate measure. We will use the latter one as the robustness measure in this paper. Some detailed discussions about the measurement of robustness are given by Nair (1992), Shoemaker, Tsui and Wu (1991), Montgomery (1991), Box and Jones (1990), Box (1988), Ryan (1988) and Kackar (1985).

[1]Design factors refer to the factors which are controllable. The concept only has meaning in contrast to the environment factors, which are hard-to-control factors.

1.2 Taguchi's Orthogonal Arrays

In robust product designs, Taguchi (Taguchi (1986); Taguchi and Phadke (1984); Taguchi and Wu (1980)) recommended the use of inner and outer orthogonal arrays to find the design factors that can be used to minimize the effect due to the variation caused by environmental sources. Most of Taguchi's inner and outer arrays are two-, three- or mixed-level classic fractional factorial designs (Kacker, Lagergren and Filliben (1990); Box, Bisgaard and Fung (1988)). In some of Taguchi's designs, the inner orthogonal arrays are highly fractional designs with complicated confounding structures, while main effects of design factors are confounding with two-factor interactions. If some interactions among design factors do indeed exist, the analysis results often are confused and misleading (Ryan (1988); Matar and Lochner (1988); Hunter (1985); Kackar (1985)). A few runs of a follow-up confirming experiment is often recommended in practice when high fractional design is used. But the confirming experiment can not detect significant design factors that fail to be found in the first place (Logothetis and Wynn (1989); Lucas (1989)).

In order to estimate the main effects of design factors free of interactions, a design with resolution IV or higher is required in the inner array. Because of the cross-array arrangement, the total number of experimental runs is the product of the size of inner and outer arrays in Taguchi's design. For a system with many design factors, the cross-array arrangement often involves a considerably large amount of experimental work.

1.3 Single Orthogonal Arrays

Many authors (Shoemaker, Tsui and Wu (1991); Montgomery (1991); Box and Jones (1990); Welch, Yu, Kang and Sacks (1990); Bullington, Hool and Maghsoodloo (1990); Lucas (1989); Bisgaard (1989); Box, Bisgaard and Fung (1988); Nair (1986)) suggested the use of a single array instead of Taguchi's cross-array in robust product designs. The use of a single orthogonal array may reduce the experimental runs and simplify the confounding structure, especially to free main effects of design factors of two-factor interactions. The single array is based on traditional fractional factorial designs. It is constructed by choosing the design generators carefully so that the effects which

Table 1: 2^{7-2}_{IV} Fractional Factorial Designs (Example 1)

Generator: $I = ABCDF = ABCEG$
Aliases: $FG = DE$

are desired are not confounded.

As a demonstration example, consider an experiment with four design factors and three environment factors.

Example 1: Suppose that three environment factors are assigned to the letters A, B, and C. Design factors are D, E, F, and G. A 2^{7-2}_{IV} fractional factorial design is given in Table 1. The design requires thirty-two runs. All main effects of design factors and design×environment interactions can be estimated.

The single array experiment plan is based on the detailed analysis of the confounding structure of resolution III or IV fractional factorial designs. An efficient design can be obtained for only certain combinations of the numbers of design and environment factors. More detailed discussions are given by Box and Jones (1990), Addelman (1962), and Whitwell and Morbey (1961).

In the following sections, we use an alternative approach to construct designs providing unbiased estimates of effects that are needed to identify significant design factors in the screen stage of the robust design of experiment. We assume that there may be some significant environment×environment and design×design interactions. All third or higher order interactions are assumed zeros. The objective is to estimate design×environment interactions and all main effects of design factors. We first investigate the general estimation capacity of cross-array design. Then we introduce block cross-array technique in our construction to reduce the required number of runs. The proposed designs are illustrated with examples. The designs are tabulated up to fifteen designs and fifteen environment factors. The generally systematic construction strategy for more than fifteen design or environment factors is described. A discussion is addressed in the last section.

Table 2: Sixty-four-run Cross-Array Design (Example 2)

$A = BD = CE$	$B = AD = CF$	$C = AE = BF$
$D = AB = EF$	$E = AC = DF$	$F = BC = DE$
$AF = BE = CD$		
$G = HK = JL$	$H = GK = JM$	$J = GL = HM$
$K = GH = LM$	$L = GJ = KM$	$M = HJ = KL$
$GM = HL = JK$		

2 What can we get from cross orthogonal arrays?

2.1 Example for Estimating Design×Environment Interactions

Consider an example with six design factors and six environment factors.

Example 2: Denote A, B, C, D, E and F as design factors and G, H, J, K, L, and M as environment factors. Suppose that we use 2_{III}^{6-3} with generators $I = ABD$, $I = ACE$ and $I = BCF$ as an inner array. Use 2_{III}^{6-3} with the generator $I = GHK$, $I = GJL$ and $I = HJM$ as an outer array. The confounding relationship of the cross-array design is in Table 2. Both inner and outer arrays require eight runs. The total number of runs is sixty-four. There is no confounding structure crossing the design factors and the environment factors. All cross interactions (design×environment interactions) can be estimated. But no main effects of design factors can be obtained unless some of the design×design interactions are assumed to be zeros.

If we replace the inner array by a sixteen-run resolution IV design, the total number of the cross design increases to 128. The inner array is a 2_{IV}^{6-2} design with generators $E = ABC$ and $F = BCD$. Main control effects are free of confounding of interactions. All design×environment interactions, as well as all main effects of design factors, can be estimated in the new cross-array design.

As mentioned before, main and interaction effects among environment factors are uncontrollable. They are of less interest in our special robust issue context. Therefore, we may use a resolution III design as the outer array. Suppose that the outer array is of resolution III. In general, if an inner array has resolution III, the cross-array design only provides the estimates of design×environment interactions. If an inner array has resolution IV, the cross-array design can be used to estimate design×environment interactions and main design factor effects. The proof of this general result about cross-array design is in Appendix I.

We can see from the above discussion that two measurements may be conducted from Taguchi's cross-array designs. One is Taguchi's SN ratio and another one is design×environment interactions. In Taguchi's design analysis, design×environment interactions are not considered. The estimation of design×environment interactions however, is sometimes more informative and provides better understanding of the system than SN ratio.

2.2 The Total Number of Runs Required in the Cross-Array Design

To decide the total number of runs in a cross-array design for different settings, we must first give a brief review of minimum runs for resolution III and IV designs.

P-B design [2] is saturated orthogonal design. However, it only exists when the number of factors is a module of four. When the number of factors investigated is not a module of four, we use a larger available P-B design. The minimum number of runs required for n factor resolution IV designs is $2n$

[2] P-B designs are obtained by selecting a subset of factor combinations from complete factorial designs. The construction of P-B designs is based on the Hadamard matrices that exist if the number of factors is a module of four (*Hedayat A. and W.D.Wallis(1978), "Hadamard matrices and their application." The Annals of Statistics*, **6**, *1184-1238*). Nongeometric properties of P-B designs cause very complicated confounding structures.

In contrast to P-B designs, 2^{n-p}_{III} fractional factorial designs with a run size of power two are not, in general, saturated designs. However, 2^{n-p}_{III} fractional factorial designs have simpler confounding structures among main and interaction effects, which provide attractive projection properties.

Table 3: Minimum Number of Experimental Runs Required in Orthogonal Designs

Number of Factors	Resolution III[1]	Resolution IV[2]
4	8	8
5	8	16
6	8	16
7	8	16
8	12	16
9	12	24
10	12	24
11	12	24
12	16	24
13	16	32
14	16	32
15	16	32

1. Fractional factorial designs with a run size of power two require more runs when the number of factors is more than 7.
2. Fractional factorial designs with a run size of power two require more runs when the number of factors is more than 8.

(Webb (1968)). A minimal resolution IV design can be obtained by folding over a resolution III design (Box and Wilson (1951)). Margolin (1969) proved that all minimal resolution IV designs must be fold-overs. The minimal number of runs required in orthogonal fractional factorial resolution III and IV designs are shown in Table 3.

To determine the number of runs in cross-array design, one needs only to get the appropriate inner and outer sizes from Table 3. The total number of required runs is the product of sizes of inner and outer arrays.

Example 3: Consider an example with six design factors and five environment factors. Because we are not interested in main effects and interactions among environment factors, we can use resolution III design in the outer array. The size of the outer array is eight. Two cases are considered separately. Case 1: We are only interested in design$\times$environment interactions. The resolution III design can be used to construct the inner array, which has eight

runs. The total number of runs is sixty-four. Case 2: We are interested in design×environment interactions and main design factor effects. In this case, a resolution IV design for the inner array is required. A minimum resolution IV design for six factors requires sixteen runs. The total number of runs in this cross-array design is 128.

It is clear to see that in case 2, the number of runs for a large-scale system can be quite high. In the next section, we introduce and implement block cross-array approach to construct the desired class of designs, in which the experimental work is significantly reduced, especially for the system with a large number of design factors.

3 Block Cross-Array Approach

3.1 A Demonstration Example

We first use a simple example to illistrate the idea of the block cross-array approach.

Example 4: Suppose that there are three design factors and two environment factors. Letters A, B, and C refer to design factors. E and F refer to environment factors. As discussed in the previous section, in order to obtain an unbiased estimation of the main effects of design factors and design×environment interactions, the inner array should be at least resolution IV and the outer array can be resolution III. Thus a total of thirty-two runs is needed in the cross-array design to estimate all three main design effects and six design×environment interactions. If we use resolution III design in both the inner and outer arrays, only design×environment interactions can be estimated. In this case, both inner and outer arrays require four runs and the cross-array consists of sixteen runs. Inner and outer arrays are in Table 4.

Let us introduce some notations. Denote the column vector associated with any factor, say factor A, as $\overline{A}$. Define $+\overline{A} = \overline{A}$ and $-\overline{A}$ be the vector

Table 4: 2^{3-1}_{III} Inner and 2^2 Outer Arrays (Example 4)

A	B	C=AB	D	E
+	+	+	+	+
+	−	−	+	−
−	+	−	−	+
−	−	+	−	−

that all signs are revised in $\overline{A}$. That is,

$$\overline{A} = \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix}$$

$$-\overline{A} = \begin{pmatrix} - \\ - \\ + \\ + \end{pmatrix}$$

Also define $\overline{I}$ be the vector of all elements +. Then $-\overline{I}$ is the vector of all elements −. Then the cross-array design for five factors A. B. C. D. and E is in Table 5. We call this cross-array *simple cross-array.*

Example 5: To describe the block fractional factorial technique. let us first construct a three-factor resolution III design 2^{3-1}_{III}. Assign C. D and E (not only D and E) to the three columns as shown in Table 6. This resolution III design is related to the outer array for the environment factors E and F. But the design has one more column for the assignment of one design factor. say. design factor C. in our example.

Next construct a resolution III (not resolution IV !) inner array for design factors A. B. and C. as in Table 7.

Obtain the cross-array for five factors A. B. C. D. E. shown in Table 8. We call this cross-array *block cross-array.* The only difference between

Table 5: Simple Cross-Array (Example 4)

A	B	C	D	E
$+\bar{I}$	$+\bar{I}$	$+\bar{I}$	$\bar{D}$	$\bar{E}$
$+\bar{I}$	$-\bar{I}$	$-\bar{I}$	$\bar{D}$	$\bar{E}$
$-\bar{I}$	$+\bar{I}$	$-\bar{I}$	$\bar{D}$	$\bar{E}$
$-\bar{I}$	$-\bar{I}$	$+\bar{I}$	$\bar{D}$	$\bar{E}$

Table 6: 2_{III}^{3-1} for Outer Array (Example 5)

C	D	E=CD
+	+	+
+	−	−
−	+	−
−	−	+

Table 7: 2_{III}^{3-1} for Inner Array (Example 5)

A	B	C=AB
+	+	+
+	−	−
−	+	−
−	−	+

Table 8: Block Cross-Array (Example 5)

A	B	C	D	E
$+\overline{I}$	$+\overline{I}$	$+\overline{C}$[1]	$\overline{D}$	$\overline{E}$
$+\overline{I}$	$-\overline{I}$	$-\overline{C}$	$\overline{D}$	$\overline{E}$
$-\overline{I}$	$+\overline{I}$	$-\overline{C}$	$\overline{D}$	$\overline{E}$
$-\overline{I}$	$-\overline{I}$	$+\overline{C}$	$\overline{D}$	$\overline{E}$

1. $\overline{C}$ is the vector corresponding to factor C in the outer array (Table 6).

this block cross-array and the simple cross-array in Table 5 is the blocks corresponding to the design factor C. In the simple cross-array, the four small blocks corresponding to the factor C is either $\overline{I}$ or $-\overline{I}$. For the block cross-array, the level arrangement of the small block $\overline{C}$ is from the outer array in Table 6. The signs before the four small blocks of $\overline{C}$ come from the settings of factor C in the inner array (Table 7). Two cross-arrays in Table 5 and Table 8 have the same number of runs. But the main design effects are confounded with design×design interactions in the simple cross-array. However, main design effects are not confounded with design×design interactions in the block cross-array. Thus, we combine two resolution III arrays (in Table 6 and Table 7) to get a cross-array that provides unbiased estimation of design×environment interactions as well as main design effects free from design×design interactions. The level settings of the block cross-array are given in Table 9. It is easy to check that the block cross-array has the desired property (in fact, this is a resolution V design).

3.2 Block Cross-Array Approach: Run Savings

The result demonstrated in the above five factor design can be extended to the general case.

Suppose that there are n design factors and m environment factors. The step by step procedure for the construction of a block cross design is as follows:

Table 9: Level Arrangement in Block Cross-Array (Example 5)

A	B	C	D	E
+	+	+	+	+
+	+	+	−	−
+	+	−	+	−
+	+	−	−	+
+	−	−	+	+
+	−	−	−	−
+	−	+	+	−
+	−	+	−	+
−	+	−	+	+
−	+	−	−	−
−	+	+	+	−
−	+	+	−	+
−	−	+	+	+
−	−	+	−	−
−	−	−	+	−
−	−	−	−	+

1. Choose the maximal possible integer p and construct a resolution III design 2^{n-p}_{III} for n design factors. Denote this design matrix as X.

2. If the design obtained in Step 1 has resolution IV or higher [3], construct a resolution III design for m environment factors. Otherwise, construct a resolution III design for m+1 factors [4]. Denote the design matrix as Z.

3. Obtain the block cross-array for these n+m factors.
Case 1. The design obtained in Step 1 has resolution IV or higher. In this case, the block cross-array is exactly same as simple cross-array and there is no run saving.
Case 2. There exists a resolution III design 2^{n-p}_{III} for some p in Step 1. Assume q factors can be accommodated in this 2^{n-p}-run design such that the design for these q factors is of resolution IV or higher [5]. Assume the first q columns in X are corresponding to these q factors. In this case, assign the first q design factors to the small block $\overline{I}$ with signs corresponding to the first q columns of X; assign the remaining n-q design factors with the small block obtained from the first column of Z with the signs corresponding to the last n-q columns of X; finally, assign m environment factors to the last m columns of Z.

We use two examples to illustrate the above general procedure.

Example 6: Consider six design factors and six environment factors: that is, n=6 and m=6. Let A, B, C, D, E and F refer to six design factors and G, H, J, K, L and M refer to six environment factors. There exists a resolution 2^{6-3}_{III} design (p=3) for six design factors. The maximal integer q is four ($q = 4$) in this example: that is, if we only consider factors A, B, C and D, the design is of resolution IV. The design matrix of X is given in Table 10. Because there exists a resolution III design 2^{6-3}_{III}, we need to construct a resolution III design for seven (m+1=7) factors. We use a eight run P-B design and get the design matrix Z. Let $\overline{R}$ be the first column of Z and $\overline{G}$.

[3] There does not exist resolution III designs for certain number of factors, such as one, two, four and eight.

[4] The design in Step 2 may not have a run size of power two. For example, we may use P-B designs.

[5] It is clear that q $\geq$ n-p or n-q $\leq$ p.

Table 10: 2^{6-3}_{III} Array X (Example 6)

A	B	C	D=ABC	E=AB	F=AC
+	+	+	+	+	+
+	+	−	−	+	−
−	+	+	−	−	−
+	−	+	−	−	+
+	−	−	+	−	−
−	−	+	+	+	−
−	+	−	+	−	+
−	−	−	−	+	+

Table 11: Block Cross-Array (Example 6)

A	B	C	D	E	F	G	H	J	K	L	M
$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{R}$	$+\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{R}$	$-\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{R}$	$-\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{R}$	$+\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{R}$	$-\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{R}$	$-\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{R}$	$+\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$
$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{R}$	$+\overline{R}$	$\overline{G}$	$\overline{H}$	$\overline{J}$	$\overline{K}$	$\overline{L}$	$\overline{M}$

...., $\overline{M}$ are the columns corresponding to factors G, ..., M in Z, respectively. The total number of runs in the block cross-array is sixty-four (8×8). The block cross design matrix is given in Table 11.

Example 7: Consider thirteen design factors and two environment factors: that is, n=13 and m=2. Let A, B, C, D, E, F, G, H, J, K, L, M and N refer to eleven design factors and O and P refer to two environment factors. There exists a resolution 2^{13-9}_{III} design ($p = 9$) for thirteen design factors. In this example, $q = 9$. The design matrix of X is given in Table 12. Because there exists a resolution III design 2^{13-9}_{III} for design factors, we need to construct a resolution III design for three (m+1=3) factors. We use a 2^{3-1}_{III}

Table 12: 2_{III}^{13-9} Array X (Example 7)

A	B	C	D	E=ABC	F=ABD	G=ACD	H=BCD	J=ABCD	K=AB	L=AC	M=AD	N=BC
+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	−	+	−	+	−	−	−	+	−	+	−
−	+	+	+	−	−	−	+	−	−	−	−	+
+	−	+	+	−	−	+	−	−	−	+	+	−
+	−	−	+	+	−	−	+	+	−	−	+	+
−	−	+	+	+	+	−	−	+	+	−	−	−
−	+	−	+	+	−	+	−	+	−	+	−	−
−	−	−	+	−	+	+	+	−	+	+	−	+
+	+	+	−	+	−	−	−	−	+	+	−	+
+	+	−	−	−	−	+	+	+	+	−	−	−
−	+	+	−	−	+	+	−	+	−	−	+	+
+	−	+	−	−	+	−	+	+	−	+	−	−
+	−	−	−	−	+	+	−	−	−	−	−	+
−	−	+	−	+	−	+	+	−	+	−	+	−
−	+	−	−	+	+	−	+	−	−	+	+	−
−	−	−	−	−	−	−	−	+	+	+	+	+

design and get the design matrix Z. Let $\overline{R}$ be the first column of Z and $\overline{O}$, $\overline{P}$ are the columns corresponding to factors O and P in Z, respectively. The total number of runs in the block cross-array is sixty-four (16×4). The block cross design matrix is given in Table 13.

Table 14 lists the results up to fifteen design and fifteen environment factors. The argument in Example 6 and 7 can be easily extended to the general block cross-array cases. The proof is in Appendix II.

Table 13: Block Cross-Array (Example 7)

A	B	C	D	E	F	G	H	J	K	L	M	N	O	P
$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{R}$	$+\overline{R}$	$+\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{R}$	$-\overline{R}$	$+\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{R}$	$-\overline{R}$	$-\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{R}$	$+\overline{R}$	$+\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{R}$	$-\overline{R}$	$+\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{R}$	$-\overline{R}$	$-\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{R}$	$+\overline{R}$	$-\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{R}$	$+\overline{R}$	$-\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{R}$	$+\overline{R}$	$-\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{R}$	$-\overline{R}$	$-\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{R}$	$-\overline{R}$	$+\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{R}$	$+\overline{R}$	$-\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{R}$	$-\overline{R}$	$-\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{R}$	$-\overline{R}$	$+\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{I}$	$-\overline{I}$	$+\overline{I}$	$-\overline{I}$	$-\overline{R}$	$+\overline{R}$	$+\overline{R}$	$-\overline{R}$	$\overline{O}$	$\overline{P}$
$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$-\overline{I}$	$+\overline{I}$	$+\overline{R}$	$+\overline{R}$	$+\overline{R}$	$+\overline{R}$	$\overline{O}$	$\overline{P}$

4 Discussion

The motivation behind our work is to use robust designs to screen a large number of design factors. In order to estimate main design effects free of confounding with interactions, a simple cross-array requires a resolution IV design as an inner array. For a large number of design factors, the number of runs in the simple cross-array is quite high and very costly. The block cross-array design combines two resolution III designs to reduce the experimental runs.

There is another possible application of the block cross-array design. It is useful if investigators are interested in finding the best settings of design factors such that the variation caused by environment change is minimum. Box and Jones (1990) proved that to achieve minimum variation one needs only to know the main and interaction effects among environment factors. If we assign environment factors in the inner array and design factors in the outer array in our proposed block cross-array designs, main environment effects and design$\times$environment interactions can be estimated. Box and Jones used single arrays to construct appropriate designs for some certain combinations of the numbers of design and environment factors. The single array approach is supposed to use less runs and to be superior to the Taguchi cross-array method. For some combinations of the numbers of design and environment factors, there is no easy way to get efficient desired designs. In this case, one must use a larger size design that is available. For example, it requires sixty-four runs to investigate eleven design and three environment factors. But it requires the same number of runs for nine or ten design factors and three environment factors (Box and Jones (1990b), pp. 41). Compared to sixty-four runs, our block cross designs need only forty-eight runs for nine or ten design factors and three environment factors. In the cases that there are single array designs available, the number of runs required in the block cross-array designs are equal to or less than those given in Box and Jones' work (see Table 14).

As a final point, the result can be extended to three- or mixed-level designs to detect the curvature and cross-interactions among design factors.

Table 14: Number of Runs in Block Cross-Array Design(BCAD) and Single Array Design(SAD)

Number of Design Factors	Number of Envir. Factors	Number of Runs in X	Number of Runs in Z	Number of Runs in BCAD	Number of Runs in SAD[1]
1	1	2	2	4	4
1	2	2	3	8	8
1	3	2	3	8	8
1	4	2	8	16	16
1	5	2	8	16	16
1	6	2	8	16	16
1	7	2	8	16	16
1	8	2	12	24	32
1	9	2	12	24	32
1	10	2	12	24	32
1	11	2	12	24	32
1	12	2	16	32	32
1	13	2	16	32	32
1	14	2	16	32	32
1	15	2	16	32	32
2	1	4	2	8	8
2	2	4	4	16	16
2	3	4	4	16	16
2	4	4	8	32	32
2	5	4	8	32	32
2	6	4	8	32	32
2	7	4	8	32	32
2	8	4	12	48	64
2	9	4	12	48	64
2	10	4	12	48	64
2	11	4	12	48	64
2	12	4	16	64	64
2	13	4	16	64	64
2	14	4	16	64	64
2	15	4	16	64	64
3	1	4	4	16	16
3	2	4	4	16	16
3	3	4	8	32	32
3	4	4	8	32	32
3	5	4	8	32	32
3	6	4	8	32	32
3	7	4	12	48	64
3	8	4	12	48	64
3	9	4	12	48	64
3	10	4	12	48	64
3	11	4	16	64	64
3	12	4	16	64	128
3	13	4	16	64	128
3	14	4	16	64	128
3	15	4	20	80	128
4	1	8	2	16	16
4	2	8	4	32	32
4	3	8	4	32	32
4	4	8	8	64	64
4	5	8	8	64	64
4	6	8	8	64	64
4	7	8	8	64	64
4	8	8	12	96	NA
4	9	8	12	96	NA
4	10	8	12	96	NA
4	11	8	12	96	NA
4	12	8	16	128	NA
4	13	8	16	128	NA
4	14	8	16	128	NA
4	15	8	16	128	NA

1 The results are based on Box and Jones (1991b)

Number of Design Factors	Number of Envir Factors	Number of Runs in X	Number of Runs in Z	Number of Runs in BCAD	Number of Runs in SAD
5,6,7	1	8	2	16	16
5,6,7	2	8	4	32	32
5,6,7	3	8	8	64	64
5,6,7	4	8	8	64	64
5,6,7	5	8	8	64	64
5,6,7	6	8	8	64	NA
5,6,7	7	8	12	96	NA
5,6,7	8	8	12	96	NA
5,6,7	9	8	12	96	NA
5,6,7	10	8	12	96	NA
5,6,7	11	8	16	128	NA
5,6,7	12	8	16	128	NA
5,6,7	13	8	16	128	NA
5,6,7	14	8	16	128	NA
5,6,7	15	8	20	160	NA
8,9,10	1	16	2	32	32
8,9,10	2	16	4	64	64
8,9,10	3	16	4	64	64
8,9,10	4	16	8	64	NA
8,9,10	5	16	8	128	NA
8,9,10	6	16	8	128	NA
8,9,10	7	16	8	128	NA
8,9,10	8	16	12	192	NA
8,9,10	9	16	12	192	NA
8,9,10	10	16	12	192	NA
8,9,10	11	16	12	192	NA
8,9,10	12	16	16	256	NA
8,9,10	13	16	16	256	NA
8,9,10	14	16	16	256	NA
8,9,10	15	16	16	256	NA
11,12,13,14,15	1	16	2	32	NA
11,12,13,14,15	2	16	4	64	NA
11,12,13,14,15	3	16	8	128	NA
11,12,13,14,15	4	16	8	128	NA
11,12,13,14,15	5	16	8	128	NA
11,12,13,14,15	6	16	8	128	NA
11,12,13,14,15	7	16	12	192	NA
11,12,13,14,15	8	16	12	192	NA
11,12,13,14,15	9	16	12	192	NA
11,12,13,14,15	10	16	12	192	NA
11,12,13,14,15	11	16	16	256	NA
11,12,13,14,15	12	16	16	256	NA
11,12,13,14,15	13	16	16	256	NA
11,12,13,14,15	14	16	16	256	NA
11,12,13,14,15	15	16	20	320	NA

Appendix I : Estimation of Design×Environment Interactions

Assumption (1): The expected response $E(y)$ can be described by the model

$$E(y) = \mu + \sum_{i=1}^{n} \alpha_i x_i + \sum_{j=1}^{m} \beta_j z_j + \sum_{i \neq j} \gamma_{ij} x_i z_j + \sum_{i \neq k} \delta_{ik} x_i x_k + \sum_{j \neq l} \eta_{jl} z_j z_l \quad (1)$$

where μ, α_i, β_j, γ_{ij}, δ_{ik} and η_{jl} are general mean, design, environment, design×environment, design×design and environment×environment effects, respectively.

Assumption (2): Let

$$X = [X_1, ..., X_n]$$

and

$$Z = [Z_1, ..., Z_m]$$

be inner and outer arrays, where vector X_i corresponds to α_i, and Z_j corresponds to β_j. All elements of X_i and Z_j are two levels, either 1 or -1, and the number of 1 and -1 are equal for each X_i or Z_j.

Assumption (3): Orthogonality

$$X_i^T X_j = 0^6$$

and

$$Z_k^T Z_l = 0$$

where $i \neq j$ and $k \neq l$.

P-B and 2_{III}^{n-p} fractional factorial designs satisfy Assumption (2) and (3).

Lemma . Under Assumption (1), (2), and (3),
(a) in the simple cross-array $X \times Z$, design×environment interactions γ_{ij} can be estimated.
(b) if design matrix

$$X = [X_1, ..., X_n]$$

[6] The product is a common vector dot product. X_i^T is the transposition of X_i.

is of resolution IV, then in the simple cross-array $X \times Z$, design$\times$environment interactions γ_{ij}s, as well as design main effects α_is are estimable.

Proof.

If $\delta_{ik} = 0$ for all $i \neq k$ and $\eta_{jl} = 0$ for $j \neq l$, then α_is and γ_{ij}s can be estimated in the simple cross-array design $X \times Z$. This follows the properties of the Kronecker product (Shoemaker, Tsui and Wu (1991)).

Part (a). In order to show that γ_{ij}s can be estimated, we need only to check that the columns corresponding to γ_{ij} and δ_{kl} are orthogonal, and the columns corresponding to γ_{ij} and η_{kl} are orthogonal in $X \times Z$. They are true because of Assumption (2).

Part (b). We need only to check furtherthat the columns corresponding to α_i and δ_{jk} are orthogonal, and the columns corresponding to α_i and η_{jk} are orthogonal. The columns corresponding to α_i and δ_{jk} are orthogonal because X is of resolution IV. The columns corresponding to α_i and η_{jk} are orthogonal because of Assumption (2).

Appendix II: Block Cross-Array Designs

Under the Assumption (1), (2), abd (3) in Appendix I,

Case 1: if the design obtained in Step 1 has resolution IV or higher, the block cross-array design is the same as the simple cross-array design. The proof is given in Appendix I (b).

Case 2: there exists a resolution III design 2^{n-p}_{III} for some p in Step 1. Suppose that q factors $X_1, \ldots, X_q$ can be accommodated in this 2^{n-p}-run design such that the design for these q factors is of resolution IV or higher. In X$\times$Z, the only difference from simple cross-array design is the settings of other n-q design factors, $X_{q+1}, \ldots, X_n$. In the block cross-array, we use the first column R in Z instead of I in the simple cross-array. However, R is orthogonal to all other columns in Z. Hence, one can easily check that all steps in Appendix I are valid in the block cross-array case.

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